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A Comparison Study of Advanced Tracking Differentiator Design Techniques

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Abstract

This paper presents a comparison study of four advanced tracking differentiators, including global robust exact differentiator, hybrid continuous nonlinear differentiator, robust exact uniformly convergent arbitrary order differentiator and Taylor expansion series based differentiator. The numerical simulations are performed by three typical signals with different type of noise to illustrate the performance of tracking differentiation, sensibility to noise and robustness ability. The results show that, over all, these differentiators could track derivate signal of low frequency signal well and Taylor expansion series based differentiator is more sensitive to noise than other three differentiators.
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1. Introduction

Tracking differentiator(TD) design has attracted much attention[1,2,3] in last two decades due to pursuing the high performance of control and navigation system. Real-time differentiators are used to solve a wide variety of problems in Aeronautic and Astronautic engineering: from the construction of feedback of aircraft control[4,5], to obtaining the high accuracy positioning and velocity[5], to the fault detection and isolation(FDI)[6], just name a few.

In practice, difference method and Kalman filter are used to obtain the derivation signals. Difference method could estimate approximately the derivatives of signals but the result is sensitive to the noise which exists in almost

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all signals. Kalman filter can reduce the disturbance and obtain the derivates signals but the model of plant must be known. Thus, it is crucial to construct a differentiator which can extract the derivate signal in noise environment and not depend on the model of plant.

The popular high-gain differentiator introduced by Kahlil [7] could track exact derivate when the gains tend to infinity which could not possible be in practice. In [8,9], differentiators was designed by slide-mode techniques. In this kind of differentiator, the upper bound for Lipschitz constant are needed. However the output of derivate estimation is not smooth because of the existing of discontinuous function. Therefore, chattering phenomenon exists in derivate estimation. In [10,11], global robust exact differentiator was designed by combining the high-gain differentiator with sliding modes differentiation though a switch function. Wang Xinhua[12] etc. proposed a hybrid continuous nonlinear differentiator in which chattering phenomenon can be reduced sufficiently. The differentiator in [13] could uniformly converge with initial differentiator error and finite-time exact convergence. In [14], a novel differentiator was built based on Taylor expansion series. Although there are many differentiator design techniques and application in literature, there is no comparison study in differentiator design techniques. In this paper the comparison study has been done to fill this gaps.

The remainder of paper is organized as follows. Section 2 introduces the problem statement and section 3 proposes four advanced tracking differentiator. Section 4 presents the numerical simulation and analysis. And finally in section 5 summarizes the paper and the main conclusions are delivered in this part.

2. Problem statement

Differentiators, in essence, are the estimators, which are not based on the model of plant. The real-time differentiators are designed to obtain the successive derivatives of given signal $\theta(t)$.

Considering a system (1)

$$\dot{x}_1 = x_2, \dot{x}_2 = x_3, \cdots\cdots, \dot{x}_n = -\lambda(t) \tag{1}$$

with input $x_1 = \theta(t)$, the problem of construction of (n-1)-th-order differentiator for $\theta(t)$ can be transformed to design estimator or observer of system (1).

3. The existing TD

In classical control theory, the differentiator is built by small time constant inertia unit. The first-order derivate of input signal U could be obtained by following linear, time-invariant, continuous-time dynamic system (2)

$$Y = \frac{s}{Ts+1}U = \frac{1}{T}\left(1 - \frac{1}{Ts+1}\right)U \tag{2}$$

where Y and U are the output and input respectively, T is the time constant and s is the Laplace operator. In fact, when time constant is small enough, the inertia unit is approximate time-delay unit. That is to say, $1/(Ts+1) \approx e^{-Ts}$.The inertia unit in system (2) could be treated as a time-delay unit with small time constant. The inverse Laplace transform of system (2) is

$$y(t) \approx \frac{1}{T}\big(u(t) - u(t-T)\big) \approx \dot{u}(t) \tag{3}$$

When signals $u(t)$ are corrupted by noise, the derivative of the usually rapidly varying noise will 'drown out' the derivative of the signals. An improved differentiator of following was proposed in Han(1999)

$$Y(s)=\frac{1}{T_2-T_1}\left(\frac{1}{T_1s+1}-\frac{1}{T_2s+1}\right)U(s) \tag{4}$$

However these two differentiators perform far from satisfactory when they face the signals with noise and high frequency signals.

To enhance the capabilities of tracking differentiators in solving the real issues, such as uncertainty, noise, disturbance, etc., several advanced techniques are proposed and briefly introduced below. The details of these advanced techniques can be founded in corresponding references.

For the sake of the simplicity and comparison, we only consider the second-order differentiator.

3.1. global robust exact differentiator(GRED)

The global robust exact differentiator(GRED) [10,11] is the weighted average of high gain differentiator and slide mode differentiator. The GRED is described as follows(5):

$$\begin{cases} y_1=\alpha x_{11}+(1-\alpha)x_{21} \\ y_2=\beta x_{12}+(1-\beta)x_{22} \end{cases} \tag{5}$$

where y_1 tracking the input $u(t)$ and y_2 tracking the derivate of input $u(t)$. In equation (5), x_{11} and x_{12} are from high gain differentiator(left part of equation (6)) while x_{21} and x_{22} are from slide mode differentiator(right part of equation (6)).

$$\begin{cases} \dot{x}_{11}=x_{12}-\frac{a_1}{\tau}\left(x_{11}-u(t)\right) \\ \dot{x}_{12}=-\frac{a_2}{\tau^2}\left(x_{11}-u(t)\right) \end{cases} \qquad \begin{cases} \dot{x}_{21}=x_{22}-\kappa_0\left|x_{21}-u(t)\right|\operatorname{sgn}\left(x_{21}-u(t)\right) \\ \dot{x}_{22}=-\kappa_1\operatorname{sgn}\left(x_{21}-u(t)\right) \end{cases} \tag{6}$$

The parameter α and β are used to switch between the two differentiator.

$$\alpha=\begin{cases} 0, & |e_1|<\varepsilon_1-c_1 \\ \frac{|e_1|-\varepsilon_1+c_1}{c_1}, & \varepsilon_1-c_1\le|e_1|<\varepsilon_1 \\ 1, & |e_1|\ge\varepsilon_1 \end{cases} \quad , \quad \beta=\begin{cases} 0, & |e_2|<\varepsilon_2-c_2 \\ \frac{|e_2|-\varepsilon_2+c_2}{c_2}, & \varepsilon_2-c_2\le|e_2|<\varepsilon_2 \\ 1, & |e_2|\ge\varepsilon_2 \end{cases} \tag{7}$$

where $e_1=x_{21}-x_{11}$, $e_2=x_{22}-x_{12}$, and $\varepsilon_1=\lambda_1\tau$, $\varepsilon_2=\lambda_2\tau$, c_1 and c_2 are the appropriate positive design parameters.

3.2. hybrid continuous nonlinear differentiator(HCND)

The hybrid continuous nonlinear differentiator(HCND) is described in [12] as follows(8):

$$\begin{cases} \dot{x}_1=x_2-k_1\left|x_1-u(t)\right|^{\frac{\alpha+1}{2}}\operatorname{sgn}\left(x_1-u(t)\right)-k_2\left(x_1-u(t)\right) \\ \dot{x}_2=-k_3\left|x_1-u(t)\right|^{\alpha}\operatorname{sgn}\left(x_1-u(t)\right)-k_4\left(x_1-u(t)\right) \end{cases} \tag{8}$$

where α , k_1 , k_2 , k_3 and k_4 are positive design parameters. $0<\alpha<1$ is selected sufficiently small and $(\alpha+1)/2\alpha$ is selected sufficiently large to obtain high tracking performance. However, α should not be too small to ensure track continuity. k_1 , k_2 , k_3 and k_4 are used to adjust the proportion between the linear differentiator and the nonlinear differentiator. In general, around the origin, the behaviour of the nonlinear differentiator to deal with large tracking error is strong. However, far from the origin, the linear differentiator behaves better when it comes large error.

3.3. robust exact uniformly convergent arbitrary order differentiator(REUCAOD)

The robust exact uniformly convergent arbitrary order differentiator(REUCAOD) is derived from high order slide mode differentiator. It is designed in [13] as follows(9):

$$\begin{cases} \dot{x}_1 = x_2 - \kappa_1\theta\left|x_1 - u(t)\right|^{\frac{1}{2}}\operatorname{sgn}\left(x_1 - u(t)\right) - k_1(1-\theta)\left|x_1 - u(t)\right|^{\frac{1+\alpha}{2}}\operatorname{sgn}\left(x_1 - u(t)\right) \\ \dot{x}_2 = -\kappa_2\theta\left|x_1 - u(t)\right|\operatorname{sgn}\left(x_1 - u(t)\right) - k_2(1-\theta)\left|x_1 - u(t)\right|^{1+\alpha}\operatorname{sgn}\left(x_1 - u(t)\right) \end{cases} \tag{9}$$

where κ_1 , κ_2 , k_1 , k_2 , θ and α are appropriate design parameters and the detailed selecting rule of these parameters can be found in [13]. The REUCAOD has two properties: finite-time exact convergence despite perturbations and uniform convergence the initial differentiation error.

3.4. Taylor series expansion based differentiator(TSEBD)

The previous three method, much like the control of dynamical system. An alternative method, which is built by Taylor Series, is given by Feng [14] as follows(10 and 11).

If $r(t)$ is a solution of differential equation with any given initial condition

$$r(t) + \varepsilon\dot{r}(t) + \frac{1}{2}\varepsilon^2\ddot{r}(t) + \frac{1}{3!}\varepsilon^3\dddot{r}(t) = u(t) \tag{10}$$

where $u(t)$ is the input, then

$$\lim_{\varepsilon\to 0}\left[\frac{3}{\varepsilon}u(t) - \frac{3}{\varepsilon}r(t) - 2\dot{r}(t) - \frac{1}{2}\ddot{r}(t)\varepsilon\right] = \dot{u}(t) \quad as\ \varepsilon\to 0\ uniformly\ in\ [a,+\infty) \tag{11}$$

However, unlike the previous three method, the TSEBD is only available to obtain 4th order derivate because this method is unstable for high order(more than 4) derivate.

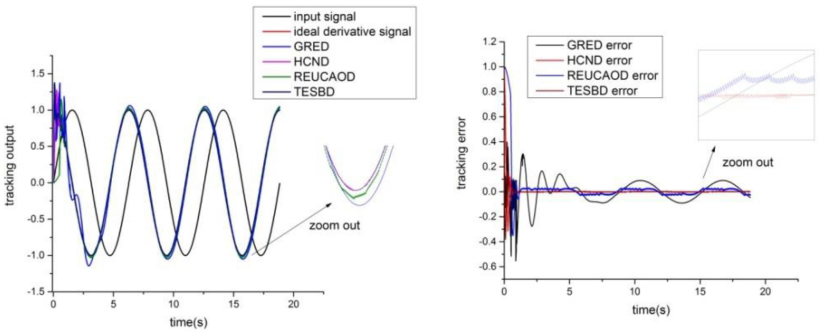
4. Numerical simulation

In this part, a numerical simulation is presented for four mentioned differentiators with three different types of input signals to illustrate their tracking performance in presence of low and high frequency signals with and without noise. The input signals is comprised by two parts, the main signal and noise. The main signal belongs to a signal set M, which contains y=sin(t), y=sin(10t) and y=t^2-t. The first two signals represent the low and high frequency respectively while last one is a delegate of polynomial signal. The noise set N is consisted by y=0, y=δ and y=0.01cos(10t), where y=δ is Gauss noise of standard normal distribution $\sim(0,1)$ which is limited by noise boundary 0.05 and y=0.01cos(10t) is the representation of high frequency noise.

The parameters in GRED are $a_1 = 0.14$, $a_2 = 0.2$, τ=0.1, κ_0=6, κ_1=28, ε_1=1, $c_1 = 0.05$, ε_2=0.5 and $c_2 = 0.05$. For HCND, $k_1 = 1$, $k_3 = 8$, $k_2 = 7$ and $k_4 = 25$ when $t \le 1$, $k_2 = 1$ and $k_4 = 8$ when $t > 1$, and α=0.2 are selected. κ_1=6.2144, κ_2=20.48, $k_1 = 7$, $k_2 = 2.1429$, $\theta = 0$ when $t \le 1$, $\theta = 1$ when $t > 1$ and α=0.06 are used in REUCAOD. In TSEBD we select ε=0.1.

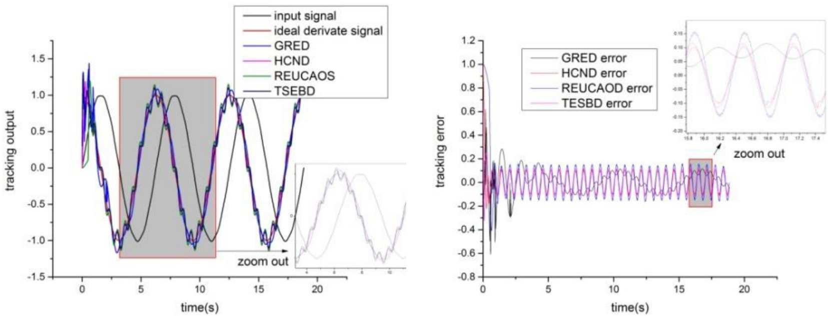
4.1. low frequency signal comparison

In the case of low frequency signal tests, the basic input signal is $y = \sin(t)$. Figure 1.a and 1.b shows the tracking output and tracking error of the basic signal. These two figure illustrate that all differentiators perform well and have roughly the same accuracy. The steady error of TSEBD is zero and the GRED have the maximum steady error. When the basic signal adds high frequency noise, as shown in figure 2.a and 2.b, the steady error of tracking differentiators are almost 20%, which illustrate that robust of these differentiator are not such satisfactory. However, figure 3.a and 3.b show that these differentiator are more sensitive to the gauss noise. The derivate tracking signal has sharp fluctuations, especially the TSEBD.



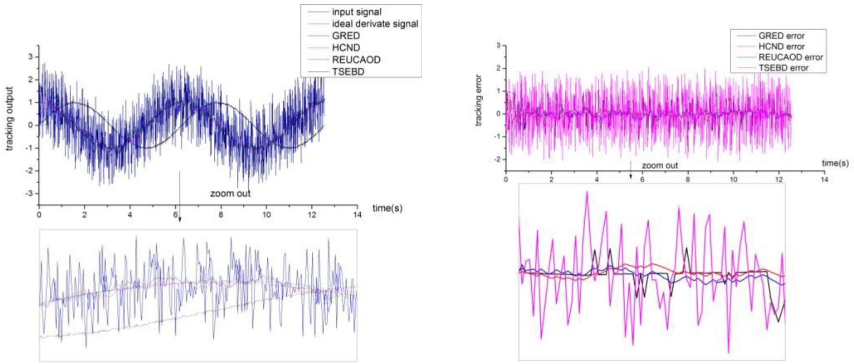
(a) (b)

Fig. 1. Differentiators tracking performance when tracking y=sin(t).(a) tracking output; (b) tracking error.



(a) (b)

Fig. 2. Differentiators tracking performance when tracking y=sin(t) with high frequency noise.(a) tracking output; (b) tracking error.

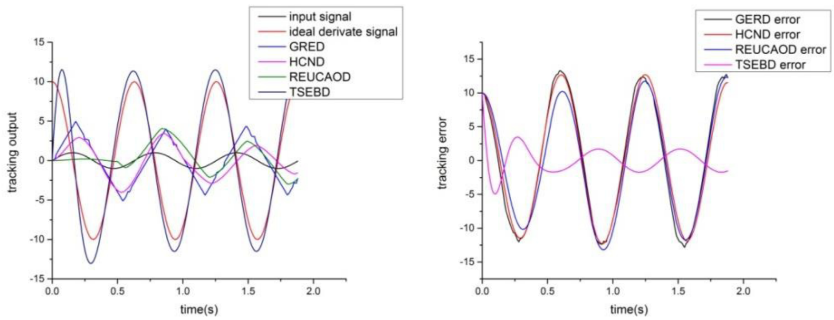


(a) (b)

Fig. 3. Differentiators tracking performance when tracking y=sin(t) with bounded gauss noise.(a) tracking output; (b) tracking error.

4.2. High frequency signal comparison

Figure 4.a, 4.b, 5.a, 5.b, 6.a and 6.b illustrate the differentiation tracking result and error of these four differentiators when basic signal $y = \sin(10t)$ without and with two typical noises. Unlike low frequency signal, these four differentiators could not tracking differentiation signal accurately in three different cases. However the TSEBD has the same trend of the ideal derivate signal.

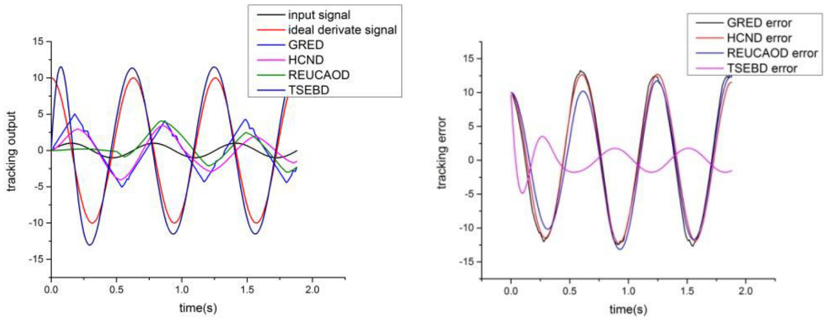


(a) (b)

Fig. 4. Differentiators tracking performance when tracking y=10sin(t).(a) tracking output; (b) tracking error.

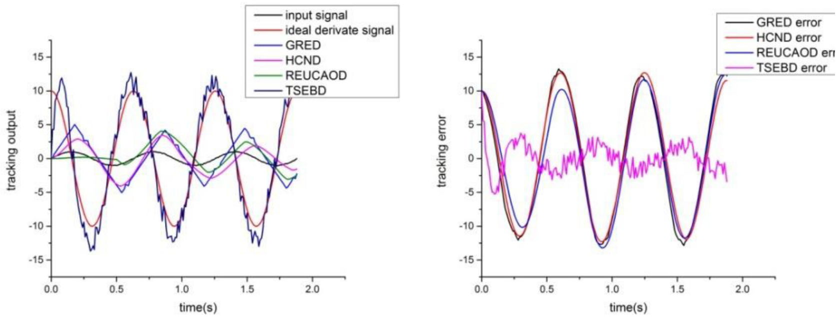
4.3. Polynomial signal comparison

As for polynomial signal tests, $y = t^2 - t$ is selected as basic input signal. Figure 7.a and 7.b show the tracking result and error of basic signal without noise. The result show that all differentiators track derivate signal well but the error of GRED is much more than other three differentiator. When these differentiators estimate the differentiation signal of basic signal with high frequency (shown in figure 8.a and 8.b), they all have almost the same estimated error. However, like the basic signal only, the GRED have the maximum error. As shown in figure 9.a and 9.b, TSEBD are more sensitive to gauss noise than other three differentiators and GRED, HCND and REUCAOD have the relative satisfactory result.



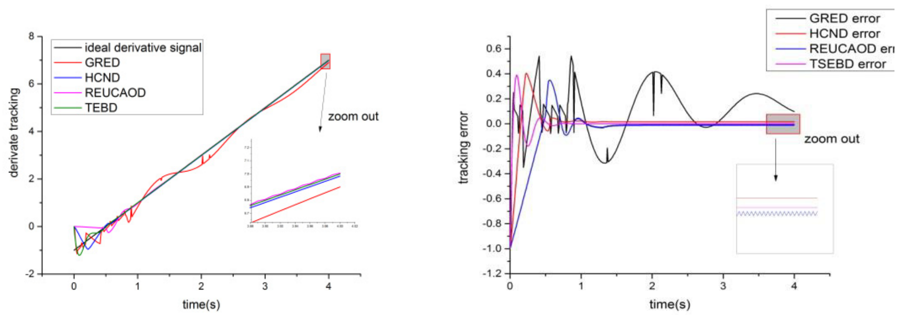
(a) (b)

Fig. 5. Differentiators tracking performance when tracking y=10sin(t) with high frequency noise.(a) tracking output; (b) tracking error.



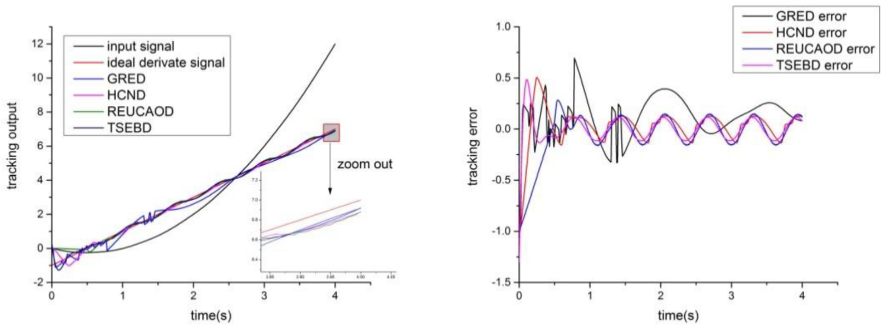
(a) (b)

Fig. 6. Differentiators tracking performance when tracking y=10sin(t) with bounded gauss noise.(a) tracking output; (b) tracking error.



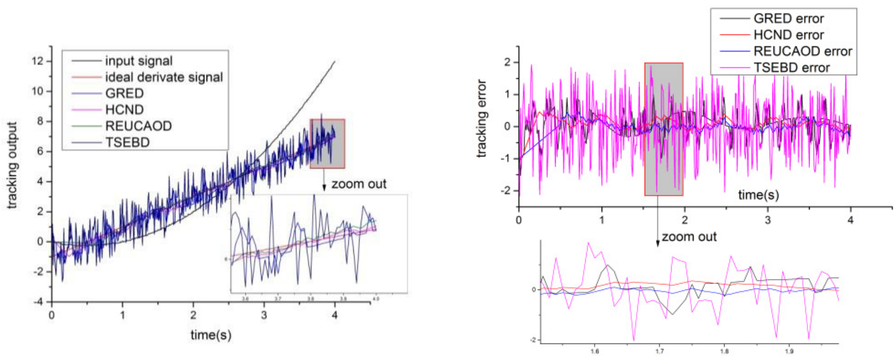
(a) (b)

Fig. 7. Differentiators tracking performance when tracking y=t^2-t.(a) tracking output; (b) tracking error.



(a) (b)

Fig. 8. Differentiators tracking performance when tracking y= y=t^2-t with high frequency noise.(a) tracking output; (b) tracking error.



(a) (b)

Fig. 9. Differentiators tracking performance when tracking y= y=t^2-t with bounded gauss noise.(a) tracking output; (b) tracking error.

5. Conclusions

In this paper, a comparison study of advanced tracking differentiator designs, including GRED, HCND, REUCAOD and TSEBD, was performed. The performance of these four differentiators are compared by simulating tests with three different signals. The main observations are made based on the simulated results as follows:

- All these four differentiators perform well when they face low frequency signal(polynomial signal can be treated as a signal with 0 frequency).
- Whether having noise or not, these four differentiators could not tracking the derivate signal of high frequency.
- TSEBD is more sensitive to noise than other three differentiators. That is to say, the robust of TSEBD is inferior to other three differentiators.
- Because of the existing of discontinuous part in GRED, HCND and REUCAOD, the chattering phenomenon happened in the steady state.
- REUCAOD has the shortest convergent time.

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